Corrigé colle d'informatique n°13.

Exercice n°1

```
> scal := (f,g) -> int(f(t)*g(t),t=0..1);

> schmidt := proc(n::integer, scal ::
  procedure)
  local i, norme, e, eps, epsprime;
  norme := f -> sqrt(scal(f,f));
  for i from 0 to n do
       e[i] := unapply(x^i,x);
  od;
  eps[0] := e[0]/norme(e[0]);
  for i from 1 to n do
      epsprime[i] := e[i] -
  sum('scal(e[i],eps[k])*eps[k]','k'=0..i-1);
      eps[i] := epsprime[i] / norme(epsprime[i]);
  od;
  return(eps);
  end;
```

schmidt := **proc**(n :: *integerscal* :: *procedure*)
local i, *norme*, e, *eps*, *epsprime*;
norme := **proc**(f)
option *operator*, *arrow*;
sqrt(*scal*(f, f))
end proc; ;
for i **from** 0 **to** n :: *integer* **do**
$e[i] := unapply(x^i, x)$
end do; ;
$eps[0] := e[0] * norme(e[0])^{-1}$;
for i **to** n :: *integer* **do**
$epsprime[i] := e[i] - sum('scal :: procedure(e[i], eps[k]) * eps[k]', 'k' = 0..i-1)$;
$eps[i] := epsprime[i] * norme(epsprime[i])^{-1}$
end do; ;
return *eps*
end proc;

```
> eps := schmidt(4,scal);
```

$$eps := eps$$

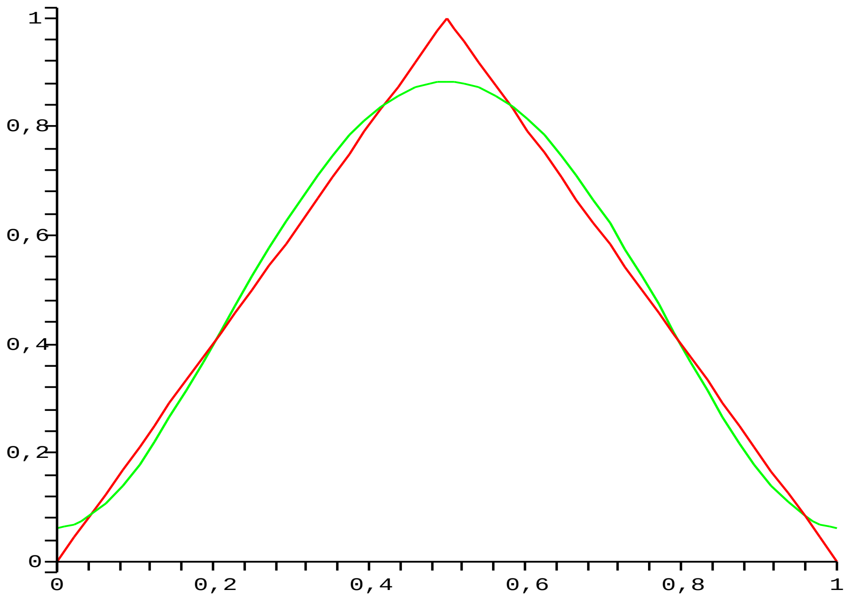
```
> f := x -> 1 - abs(2*x-1);
```

$$f := x \mapsto 1 - |2\,x - 1|$$

```
> g := sum('scal(f,eps[i])*eps[i]','i'=0..4);
```

$$g := 1/16\,e_0 + \frac{105}{8}\,e_2 + \frac{105}{8}\,e_4 - \frac{105}{4}\,e_3$$

```
> plot({f,g},0..1);
```



Exercice n°2

```
> scal := (P,Q) -> sum('P(l/10)*Q(l/10)','l'=0..10):

> eps := schmidt(7,scal);
```

$$eps := eps$$

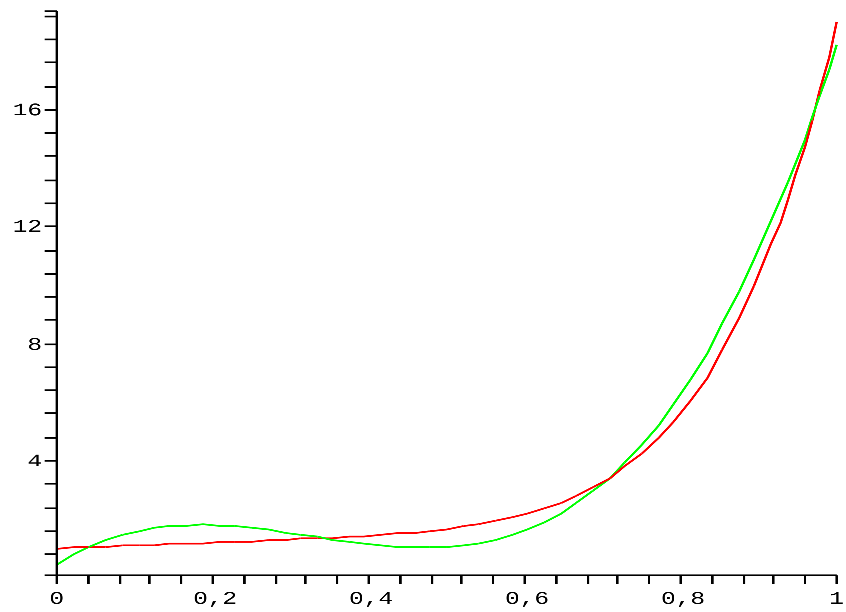
```
> P := x -> 12*x^7+5*x^6+x+1;
```

$$P := x \mapsto 12\,x^7 + 5\,x^6 + x + 1$$

```
> Q := sum('scal(P,eps[i])*eps[i]','i'=0..3);
```

$$Q := \frac{24841}{55000}\,e_0 + \frac{841839}{50000}\,e_1 - \frac{6986081}{110000}\,e_2 + \frac{443252}{6875}\,e_3$$

```
> plot(P,Q,0..1);
```



Exercice n°3

```
> f := (a,b) -> int((sinh(x)-a*x-b)^2,x=0..1);
```

$$f := (a,b) \mapsto \int_0^1 (\sinh(x) - ax - b)^2 dx$$

première méthode : on utilise les fonctions évoluées de Maple

```
> minimize(f(a,b),location)[2];
```

$$\left\{[\{b = -2\cosh(1) + 6\sinh(1) - 4, a = 6 + 6\cosh(1) - 12\sinh(1)\}, -9/2 + \frac{25}{2}\cosh(1)\sinh(1) - 4(\cosh(1))^2 [-12(\sinh(1))^2 - 4\cosh(1) + 12\sinh(1)]\right\}$$

Deuxième méthode : on reconnait une distance au carré pour le produit scalaire de l'exercice 1. On projette et on détermine a et b.

```
> scal := (f,g) -> int(f(t)*g(t),t=0..1);
```

```
> eps := schmidt(1,scal);
```

$$eps := eps$$

```
> g := sum('scal(sinh,eps[i])*eps[i]','i'=0..1);
```

$$g := (\cosh(1) - 1)e_0 + 2\left(\sqrt{3} + 3/2\sqrt{3}e^{-1} - 1/2\sqrt{3}e^1\right)(e_1 - 1/2\,e_0)\sqrt{3}$$

```
> expand(g(t));a := expand(coeff(g(t),t,1)); b := expand(coeff(g(t),t,0)); f(a,b);
```

$$\cosh(1) - 4 + 6\,t + 9\,e^{-1}t - 9/2\,e^{-1} - 3\,e^1 t + 3/2\,e^1$$

$$a := 6 + 9\,e^{-1} - 3\,e^1$$

$$b := \cosh(1) - 4 - 9/2\,e^{-1} + 3/2\,e^1$$

$$-4\cosh(1) + 12\sinh(1) + 1/2\cosh(1)\sinh(1) - (\cosh(1))^2 + \frac{27}{4}e^{-2} - 9\cosh(1)e^{-1} + 3\cosh(1)e^1 + 3/4\,e^2 - 9 + 18\,e^{-1}\sinh(1) - 6\,e^1\sinh(1)$$

Autre méthode possible : le calcul différentiel

```
> a:='a': b := 'b':
> solve(diff(f(a,b),a),diff(f(a,b),b),a,b);
```

$$\{a = 6 + 6\cosh(1) - 12\sinh(1), b = -2\cosh(1) + 6\sinh(1) - 4\}$$

Exercice n°4

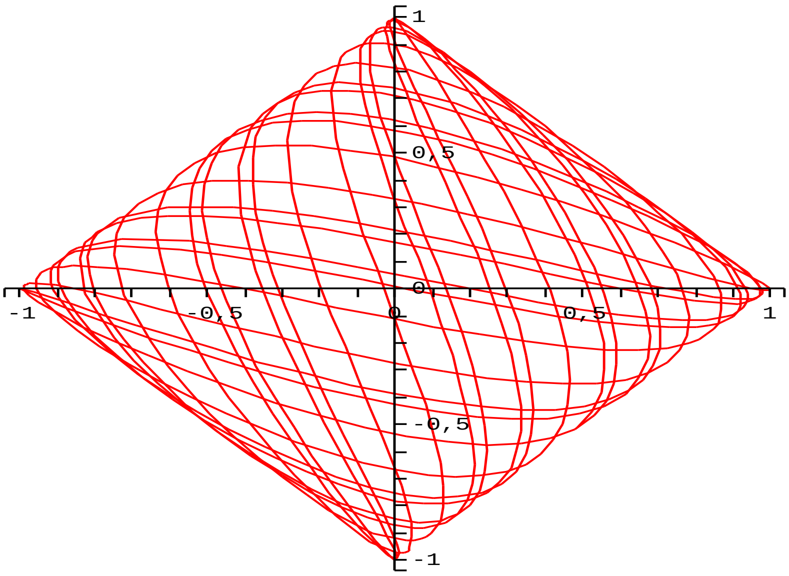
```
> restart:
> dsolve({diff(x(t),t$2)+omega^2*x(t)-K*omega^2*y(t),
>       diff(y(t),t$2)+omega^2*y(t)-K*omega^2*x(t),
>       x(0)=0,D(x)(0)=0,D(y)(0)=0,y(0)=1},{x(t),y(t)});
```

$$\left\{y\left(t\right) = 1/2\,\cos\left(\omega\,\sqrt{K+1}t\right) + 1/2\,\cos\left(\omega\,\sqrt{-K+1}t\right), x\left(t\right) = -1/2\,\cos\left(\omega\,\sqrt{K+1}t\right) + 1/2\,\cos\left(\omega\,\sqrt{-K+1}t\right)\right\}$$

```
> assign(%);
x := unapply(x(t),t,K,omega);
y := unapply(y(t),t,K,omega);
```

$$x := (t, K, \omega) \mapsto -1/2\,\cos\left(\omega\,\sqrt{K+1}t\right) + 1/2\,\cos\left(\omega\,\sqrt{-K+1}t\right)$$
$$y := (t, K, \omega) \mapsto 1/2\,\cos\left(\omega\,\sqrt{K+1}t\right) + 1/2\,\cos\left(\omega\,\sqrt{-K+1}t\right)$$

```
> plot([x(t,0.5,10),y(t,0.5,10),t=0..10]);
```

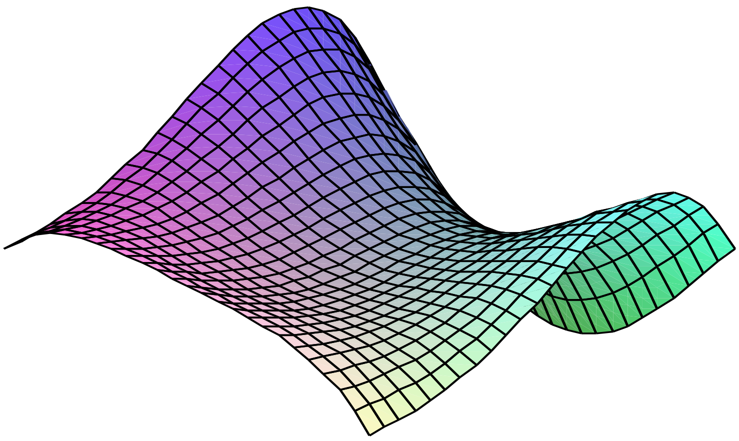


Exercice n°5 :

```
> f := (x,y) -> 2*x*(1-x^2-y^2)-x^4-y^4;
```

$$(x, y) \mapsto 2\ (x) \left(1 - x^2 - y^2\right) - x^4 - y^4 \ := \ (x, y) \mapsto 2\ (x) \left(1 - x^2 - y^2\right) - x^4 - y^4$$

```
> plot3d(f(x,y),x=-2..0,y=-1..1);
```



```
> res := solve(diff(f(x,y),x),diff(f(x,y),y),x,y);
```

$$res := \{x = 1/2, y = 0\}, \{x = -1, y = 0\}, \{x = -1, y = 0\}, \left\{x = -1/2, y = RootOf\left(2\_Z^2 - 1, label = \_L5\right)\right\},$$
$$\left\{x = -RootOf\left(\_Z^2 - \_Z - 1, label = \_L6\right), y = RootOf\left(-RootOf\left(\_Z^2 - \_Z - 1, label = \_L6\right) + \_Z^2, label = \_L7\right)\right\}$$

```
> allvalues(res[4]);allvalues(res[5]);
```

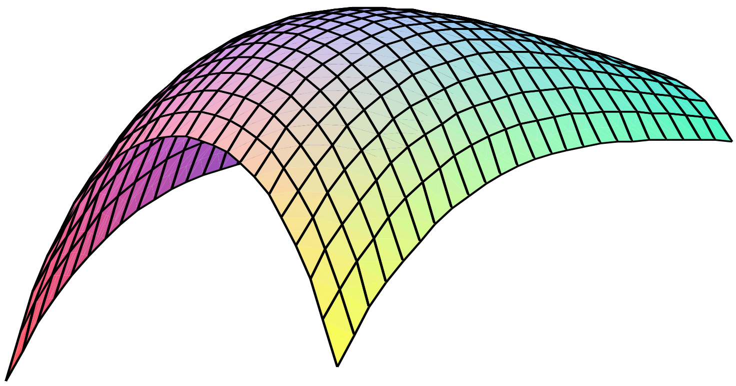
$$\left\{x = -1/2, y = 1/2\sqrt{2}\right\}, \left\{x = -1/2, y = -1/2\sqrt{2}\right\}$$

$$\left\{y = 1/2\sqrt{2 + 2\sqrt{5}}, x = -1/2\sqrt{5} - 1/2\right\}, \left\{y = -1/2\sqrt{2 + 2\sqrt{5}}, x = -1/2\sqrt{5} - 1/2\right\},$$
$$\left\{y = 1/2\sqrt{2 - 2\sqrt{5}}, x = 1/2\sqrt{5} - 1/2\right\}, \left\{y = -1/2\sqrt{2 - 2\sqrt{5}}, x = 1/2\sqrt{5} - 1/2\right\}$$

```
> expand(f(1/2+u,v));plot3d(f(x,y),x=0..1,y=-1..1);
```

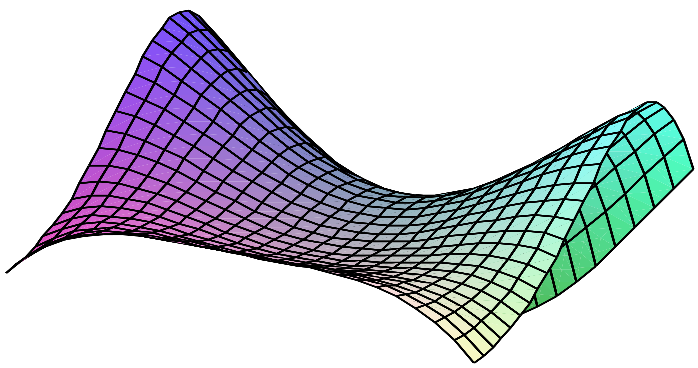
$$\frac{11}{16} - 9/2\,u^2 - v^2 - 4\,u^3 - 2\,uv^2 - u^4 - v^4$$

On a bien un extremum en $(1/2, 0)$.



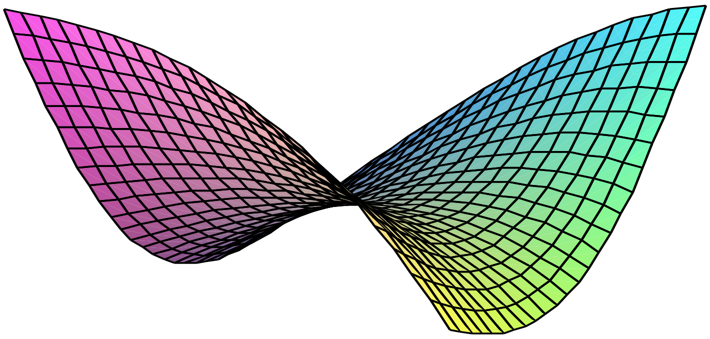
```
> expand(f(-1+u,v));plot3d(f(x,y),x=-2..0,y=-1..1);# on n'a pas d'extremum en (-1,0)
```

$$2\,v^2 + 2\,u^3 - 2\,uv^2 - 1 - u^4 - v^4$$



```
> expand(f(-1/2+u,1/sqrt(2)+v));plot3d(f(x,y),x=-1..0,y=0.5..1);#
  on n'a pas d'extremum en (-1/2,1/sqrt(2))
```

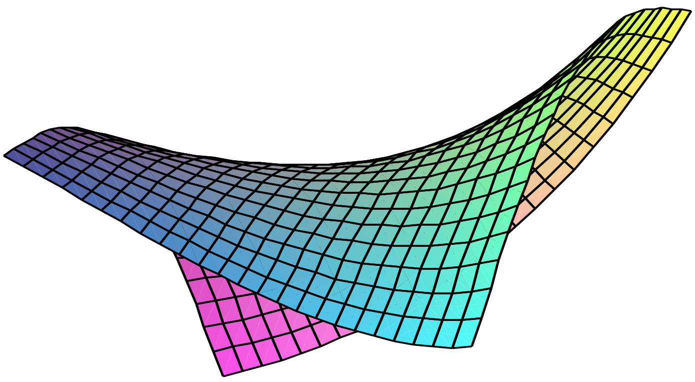
$$-\frac{9}{16} + 3/2\,u^2 - 2\,v^2 - 2\,u\sqrt{2}v - 2\,uv^2 - u^4 - 2\,\sqrt{2}v^3 - v^4$$



```
> expand(f(-1/2+u,-1/sqrt(2)+v));plot3d(f(x,y),x=-1..0,y=0.5..1);
```

$$-\frac{9}{16} + 3/2\,u^2 - 2\,v^2 + 2\,u\sqrt{2}v - 2\,uv^2 - u^4 + 2\,\sqrt{2}v^3 - v^4$$

On constate que l'on n'a pas d'extremum en $(-1/2, 1/\sqrt(2))$.



Il faut envisager tous les points déterminés dans la première partie de l'exercice.

Exercice n°6

```
> with(geom3d):
```

```
Warning, the name polar has been redefined
```

```
> point(A,1,2,3); point(B,-3,4,5); point(C,3,-4,5); point(D,-3,4,-5);
```

$$A$$
$$B$$
$$C$$
$$D$$

```
> AreCoplanar(A,B,C,D); # ils ne sont pas
  coplanaires
```

$$\mathit{false}$$

```
> sphere(s, [A, B, C, D], 'centername'=m );
```

$$s$$

```
> detail(s);
```

```
> radius(s); # rayon de la sphère
```

$$\sqrt{131}$$

```
> detail(center(s)); # coordonnées du centre
```

Pour trouver l'équation du cercle circonscrit au triangle ABC, il suffit de prendre l'intersection entre la sphère précédente et le plan contenant les trois points A, B et C.

```
> plane(p,[A,B,C],[x,y,z]);
```

$$p$$

```
> EquationSphere := Equation(s,[x,y,z]);
```

$$\mathit{EquationSphere} := -50 + x^2 + y^2 + z^2 + \tfrac{72}{5}\,x + \tfrac{54}{5}\,y = 0$$

```
> EquationPlan := Equation(p);
```

$$\mathit{EquationPlan} := -100 + 16\,x + 12\,y + 20\,z = 0$$

```
> intersection(c,p,s); # Ca ne marche pas
```

```
Error, (in intersection) wrong type of arguments
```

```
> solve(EquationSphere,EquationPlan,x,y,z);
```

$$\{y = y, z = 5 - 4/5\,x - 3/5\,y, x = 1/5\,\mathit{RootOf}\left(-625 + \_Z^2 + 34\,y^2 - 200\,x + 120\,y + 16\,x^2 + 24\,(x)\,(y) + 72\,\_Z, \mathit{label} = \_L10\right)\}$$

```
> allvalues(%);
```

$$\left\{x = -\frac{36}{5} + 1/5\,\sqrt{1921 - 34\,y^2 + 200\,x - 120\,y - 16\,x^2 - 24\,(x)\,(y)}, y = y, z = 5 - 4/5\,x - 3/5\,y\right\},$$
$$\left\{x = -\frac{36}{5} - 1/5\,\sqrt{1921 - 34\,y^2 + 200\,x - 120\,y - 16\,x^2 - 24\,(x)\,(y)}, y = y, z = 5 - 4/5\,x - 3/5\,y\right\}$$